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Zonostrophic turbulence

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Abstract

Geostrophic turbulence is a flow regime attained by turbulent, rotating, stably stratified fluids in near-geostrophic balance. When a small-scale forcing is present, flows in this regime may develop an inverse energy cascade. Geostrophic turbulence has been used in geophysical fluid dynamics as a relatively simple model of the large-scale planetary and terrestrial circulations. When the meridional variation of the Coriolis parameter (or a β-effect) is taken into account, the horizontal flow symmetry breaks down giving rise to the emergence of jet flows. In a certain parameter range, a new flow regime comes to life. Its main characteristics include strongly anisotropic kinetic energy spectrum and slowly evolving systems of alternating zonal jets. This regime is a subset of geostrophic turbulence and has been coined zonostrophic turbulence; it can develop both on a β-plane and on the surface of a rotating sphere. This regime was first discovered in computer simulations but later revealed in the laboratory experiments, in the deep terrestrial oceans, and on solar giant planets where it is believed to be the primary physical mechanism responsible for the generation and maintenance of the stable systems of alternating zonal jets. The hallmarks of zonostrophic turbulence are the anisotropic inverse energy cascade and complicated interaction between turbulence and Rossby–Haurwitz waves. Addressing the goals of the conference ‘Turbulent Mixing and Beyond’ that took place in August 2007 in Trieste, Italy, this paper exposes the regime of zonostrophic turbulence to a wide scientific community, provides a survey of this regime, elaborates its main characteristics, offers novel approaches to describe and understand this phenomenon, and discusses its applicability as a model of the large-scale planetary and terrestrial circulations.

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(Some figures in this article are in colour only in the electronic version.)

1. Introduction: the route from geostrophic to zonostrophic turbulence

Since its introduction by Charney (1971), the notion of geostrophic turbulence has become a paradigm of large-scale planetary and oceanic macroturbulence (Held 1999, Read 2001, Rhines 1979, Salmon 1998, Vallis 2006). Being established as a consequence of the Taylor–Proudman theorem in rapidly rotating systems, the regime of geostrophic turbulence pertains to a chaotic nonlinear motion of stably stratified fluids in near-geostrophic balance. The basic equation describing this regime is the quasi-geostrophic, three-dimensional (3D), inviscid conservation equation for the potential vorticity q (Salmon 1998),

\[
\frac{Dq}{Dt} = \frac{\partial q}{\partial t} + J(\psi, q) = 0,
\]

where

\[
q = \nabla^2 \psi + f + \frac{\partial}{\partial z} \left[ \frac{f_0^2}{N^2(z)} \frac{\partial \psi}{\partial z} \right].
\]

Here,

\[
J(\psi, q) = \frac{\partial \psi \partial q}{\partial x \partial y} - \frac{\partial \psi \partial q}{\partial y \partial x}
\]

is the horizontal Jacobian, \(\nabla = (\partial/\partial x, \partial/\partial y)\) is the horizontal gradient operator, \(\psi\) is the geostrophic stream function introduced such that \((u, v) = (-\partial \psi / \partial y, \partial \psi / \partial x)\), and \(u\) and \(v\) are the horizontal velocity components, \(N(z)\) is the prescribed
background Brunt–Väisälä frequency, $f = f_0 + \beta y$ is the Coriolis parameter, $f_0 = 2\Omega \sin \phi_0$ is the reference Coriolis parameter at the latitude $\phi_0$, $\Omega$ is the angular velocity of the planetary rotation, $\beta$ is the meridional gradient of the Coriolis parameter (Pedlosky 1987), and $x$ and $y$ are the zonal (east–west) and meridional (north–south) coordinates, respectively. The ratio, $Pr = f_0/N$, is sometimes referred to as the Prandtl ratio (Vallis 2006). The first, second and third terms on the right-hand side of (2) represent the relative, the planetary, and the potential vorticities, respectively; the latter is associated with the isopycnal displacements.

Charney (1971) was the first to observe that equation (2) is analogous to an equation describing purely 2D flow. Furthermore, setting $f$ and $N$ constant, Vallis (2006) showed that with periodic lateral boundary conditions or the condition of zero normal flow, equation (2) possesses two quadratic invariants of the motion, the energy and the enstrophy. The analogy with 2D flows is further exemplified by rescaling the vertical coordinate by $Pr^{-1}$ and letting $\tilde{z} = z/Pr$. Vallis (2006) shows that the energy and enstrophy invariants become almost identical to those in 2D flows,

$$\tilde{E} = \int |\nabla_3 \psi|^2 \, dV, \quad \tilde{Z} = \int (\nabla_3^2 \psi)^2 \, dV,$$

(4)

where the integration is over the entire domain $V$ and $\nabla_3 = (\partial/\partial x, \partial/\partial y, \partial/\partial \tilde{z})$.

The presence of the two invariants of the motion stipulates the affinity between the dynamics of the 3D geostrophic turbulence and purely 2D turbulence including the existence of the direct enstrophy and inverse energy cascade ranges with the characteristic $-3$ and $-5/3$ spectra, respectively. Unlike the 2D turbulence, however, the inverse energy cascade in geostrophic turbulence is 3D with the vertical wavenumber scaled by the Prandtl ratio. Thus, the inverse cascade in this case sends the energy both to larger horizontal and vertical scales, the latter known as the process of barotropization (Vallis 2006).

If the flow is isotropic in $(x, y, \tilde{z})$ coordinates then, in $(x, y, z)$ coordinates, the ratio of the vertical to the horizontal scale of an eddy is of the order of the Prandtl ratio. The maximum vertical scale to which this scaling argument can be applied is $H$ giving rise to the horizontal scale

$$L_D = NH/f,$$

(5)

known as the internal Rossby deformation radius (Salmon 1998).

With the boundary conditions $\partial \psi / \partial z = 0$ at $z = 0, H$, Salmon (1998) showed that the stream function $\psi(x, y, z, t)$ admits the vertical mode expansion,

$$\psi(x, y, z, t) = \sum_{k,n} \psi_{k,n}(x, y, t) \exp(i \mathbf{k} \cdot \mathbf{r}) \cos \left( \frac{n\pi z}{H} \right),$$

(6)

where $\psi_{k,n}(x, y, t)$ is the horizontal Fourier transform of the stream function; $\mathbf{r} = (x, y)$; $\mathbf{k} = (k_x, k_y)$ is the horizontal wavenumber vector, and $n$ is the number of the vertical modes whose respective wavenumber is

$$k_n = \frac{1}{L_D} = \frac{n \pi f}{NH},$$

(7)

where $L_D^0$ being the deformation radius corresponding to the $n$th vertical mode. The mode with $n = 0$ or $L_D^0 = \infty$ corresponds to the barotropic or vertically averaged mode. The modes with $n > 0$ are known as $n$th baroclinic modes.

Using a simplified two-layer representation comprising only two modes, barotropic and baroclinic, Salmon (1998) showed that if a flow is forced on large scales, either mechanically or thermally, then the constraints of the energy and potential enstrophy conservation cause the energy to flow to smaller scales in the baroclinic mode. At scales around the deformation radius, $L_D$, due to the baroclinic instability, part of the energy flux is redirected into the barotropic mode. Since the barotropic mode exhibits properties of 2D turbulence, the energy released by the baroclinic instability on scales $O(L_D)$ feeds the inverse cascade towards the scales larger than $L_D$. If $L_D$ is much smaller than the horizontal scale of the system, $L$, i.e. the Burger number, $Bu = (L_D/L)^2$, is small, then the barotropic mode can accumulate considerable amount of energy thus exhibiting the mentioned earlier tendency to barotropization. This tendency, as well as its evidence in the oceanic and other data were discussed by Rhines (1979), Vallis (2006) and in numerous other studies. Schematically, the process of the barotropization is sketched in figure 1.

The processes in the barotropic mode when $Bu \ll 1$ are at the focus of this study. Since the barotropic mode exhibits features of 2D turbulence, the large separation between the forcing and the system scales dictated by the condition $Bu \ll 1$ facilitates the development of the inverse energy cascade over scales between $L_D$ and $L$. If the planetary rotation is fast enough, the inverse cascade becomes affected by the meridional variation of the Coriolis parameter, or a $\beta$-effect. As a result, the large-scale flow becomes strongly anisotropic and attains a regime of zonostrophic turbulence introduced by Galperin et al (2006). This regime is distinguished by a highly energetic barotropic mode of circulation. The quasi-barotropic flows associated with this circulation develop a strongly anisotropic kinetic energy spectrum whose zonal mode alone may hold more energy than all other modes combined (Chekhlov et al 1996, Galperin et al 2006, Huang et al 2001). The most distinguished visual feature of zonostrophic turbulence is the formation of a slowly changing system of alternating zonal jets spanning the entire flow domain. The emergence of such jet systems will be referred to as zonation. The zonation cannot sustain...
The route to zonostrophic turbulence

Figure 2. Schematic representation of physical processes leading from geostrophic to zonostrophic turbulence.

itself without a small-scale forcing that feeds the inverse energy cascade. The crucial role of the small-scale forcing for the process of zonation was emphasized in computer simulation studies (Galperin et al. 2006, Sukoriansky et al. 2002) and in the experimental investigations conducted in the Coriolis turntable (Read et al. 2004, 2007). Furthermore, Galperin et al. (2001), Sukoriansky et al. (2002) and Galperin et al. (2004) identified the zonostrophic regime as a basic mechanism of generation and maintenance of zonal jets in the atmospheres of giant planets and in the deep terrestrial oceans.

Figure 2 provides schematic representation of the transition from geostrophic to zonostrophic turbulence as well as the main features of the latter regime.

Due to its specific physical nature, the regime of zonostrophic turbulence lends itself to studies using a simplified barotropic vorticity equation on a β-plane or on the surface of a rotating sphere. The latter approach will be utilized in this paper.

In the next section, a brief survey of the quasi-2D turbulence with a β-effect will be given while the following section will describe the emergence of the regime of zonostrophic turbulence in various environments. The last section will provide conclusions.

2. Quasi-2D turbulence with a β-effect

Models involving quasi-2D turbulent flows on a β-plane or on the surface of a rotating sphere have been utilized for studies of the large-scale planetary and terrestrial circulations. Various aspects of the turbulence—Rossby wave interaction and anisotropization of the upscale energy transport were investigated in the pioneering studies by Newell (1969), Rhines (1975) and Holloway and Hendershot (1977). The following up studies by Vallis and Maltrud (1993), Chekhlov et al. (1996), Smith and Waleffe (1999), Huang et al. (2001), Sukoriansky et al. (2002), Galperin et al. (2006), Nadiga (2006), Scott and Polvani (2007), Sukoriansky et al. (2007) and others considered flows with small-scale forcing necessary to maintain the inverse energy cascade.

When a mechanism of the large-scale dissipation is present, the balance between the forcing and the dissipation facilitates the establishing of a steady state which may attain various flow regimes. The classification of these regimes was given by Sukoriansky et al. (2007) using long-term simulations utilizing small-scale forced barotropic vorticity equation with a linear drag on the surface of a rotating sphere. An abridged version of this classification is presented below.

We consider a 2D barotropic vorticity equation on the surface of a rotating sphere,

$$\frac{\partial \zeta}{\partial t} = -J(\psi, \zeta + f) + \nu \nabla^2 \zeta - \lambda \zeta + \xi,$$  
(8)

where \(\zeta\) is the vorticity, \(\psi\) is the stream function defined as \(\nabla^2 \psi = \zeta, \ f = 2\Omega \sin \theta\) is the Coriolis parameter (the ‘planetary vorticity’), \(\Omega\) is the angular velocity of the sphere’s rotation, \(\nu\) is the hyperviscosity coefficient, \(p\) is the power of the hyperviscous operator (\(p\) was either 4 or 8 in this study), \(\lambda\) is the linear friction coefficient, and \(\xi\) is the small-scale forcing, respectively. The forcing acts around the wavenumber \(n_E\). The Jacobian, \(J(\psi, \zeta + f)\), represents the nonlinear term, \(J(A, B) = (R^2 \cos \theta)^{-1} (A_A B_B - A_B B_A)\), where \(R\) is the sphere radius and \(\phi\) and \(\theta\) are the longitude and latitude, respectively. For convenience, the unit of length is set to be the radius of the sphere. In these units, \(R = 1\) and will be omitted in the forthcoming derivations. The natural timescale is \(T = \Omega^{-1}\) such that \(\Omega = 1\). To explore the dependency on \(\Omega\), in some of the experiments \(T\) was kept fixed while \(\Omega\) was varied.

The stream function in equation (8) can be represented via spherical harmonics decomposition,

$$\psi(\mu, \phi, t) = \sum_{m=0}^{N} \sum_{n=-m}^{n} \psi_m^m(t) Y_n^m(\mu, \phi),$$  
(9)

where \(Y_n^m(\mu, \phi)\) are the spherical harmonics (the associated Legendre polynomials); \(\mu = \sin \theta\), \(n\) and \(m\) are the total and zonal wavenumbers, respectively, and \(N\) is the total truncation wavenumber. Conventionally, the indices \(n, m\) and \(N\) are nondimensional. However, when they appear in equations below, the wavenumbers \(n\) and \(m\) have the dimension of the inverse length. Since in the units chosen \(R = 1\), we shall not differentiate between the indices and wavenumbers.

In the unforced, nondissipative, linear limit, equation (8) gives rise to Rossby–Haurwitz waves (RHW) whose dispersion relation is

$$\omega_{m,n} = -2\beta \frac{m}{n(n+1)},$$  
(10)

where \(\beta = \Omega/R\) (Holton 2004). By retaining \(\beta\) in the forthcoming equations, we preserve the transparency between the cases of a β-plane and rotating sphere.

In the forced, dissipative regime, the balance between the small-scale forcing and large-scale dissipation gives rise to a steady state. The kinetic energy spectrum in that steady state can be calculated as

$$E(n) = \sum_{m=-n}^{n} E(n, m) = \frac{n(n+1)}{4} \sum_{m=-n}^{n} (|\psi_m^m|^2),$$  
(11)

where the modal spectrum, \(E(n, m)\), is the spectral energy density per mode \((n, m)\), and the angular brackets indicate an ensemble or time average (Boer 1983, Boer and Shepherd 1983). The spectrum \(E(n)\) can be represented as a sum of the
zonal and nonzonal, or residual components, \( E(n) = E_Z(n) + E_R(n) \), where the zonal spectrum is \( E_Z(n) = E(n, 0) \).

As elaborated by Sukoriansky et al. (2007), the large-scale drag absorbs the upscale propagation of the inverse energy cascade and causes the flattening of both zonal and residual spectra at some wavenumber \( n_R \) which can be identified as a friction wavenumber. The magnitude of \( n_R \) depends on \( \lambda, \beta \) and \( \epsilon \). Another important parameter in such flows is the wavenumber \( n_\beta \propto (\beta^3/\epsilon)^{1/3} \). It was derived by Vallis and Maltrud (1993) for flows on a \( \beta \)-plane by equating the Rossby wave period with the turbulent eddy turnover time. Sukoriansky et al. (2007) further elucidated that \( n_\beta = 0.5(\beta^3/\epsilon)^{1/3} \) characterizes the threshold of the inverse cascade anisotropization (see below). Finally, the parameter \( n_R = (\beta/2U)^{1/2} \) \( U \) being the rms fluid velocity, is the Rhines wavenumber which is close to \( n_R \) in the zonostrophic regime.

The interrelationship between \( n_R, n_\beta \) and \( n_\epsilon \) determines the nature of the steady-state flow regime on the surface of a rotating sphere. Using an extensive series of long-term steady-state simulations, Sukoriansky et al. (2007) identified four possible flow regimes in the parameter space \((n_\beta, n_R)\) as shown in figure 3. These regimes differ from each other by the degree of anisotropy and the nature of the wave–turbulence interaction. Two of these regimes, friction-dominated and zonostrophic, harbor interesting physics and will be considered in more details. The other two regimes are intermediate and nonuniversal and are of lesser importance for the present study.

The friction-dominated regime occupies the subspace to the left of the top dashed line in figure 3 and is delineated by the inequality \( R_\beta \gtrsim 1.5 \). Figure 4 shows the total and the modal energy spectra typical of this regime. In agreement with Huang et al. (2001) and Sukoriansky et al. (2002), \( E(n) \) in this case closely follows the classical Kolmogorov–Batchelor–Kraichnan (KBK) scaling,

\[
E(n) = C_K \epsilon^{2/3} n^{-5/3}, \quad C_K \simeq 6. \tag{12}
\]

For large \( n \), the modal spectrum is given by

\[
E(m, n) = C_K \epsilon^{2/3} n^{-5/3}/(2n + 1) \simeq (1/2)C_K \epsilon^{2/3} n^{-8/3} \tag{13}
\]

for every \( m \) including \( m = 0 \). For \( n < n_\beta \), figure 4 reflects the \( \beta \)-effect caused anisotropization of the inverse energy cascade and preferential energy flux into low-\( m \) modes.

In the zonostrophic regime, the zonal and residual spectra are given by

\[
E_Z(n) = C_Z \epsilon^{2/3} n^{-5/3}, \quad C_Z \sim 0.5, \tag{14a}
\]

\[
E_R(n) = C_K \epsilon^{2/3} n^{-5/3}, \quad C_K \sim 5–6, \tag{14b}
\]

thus pointing to a strong spectral anisotropy. On the surface of a rotating sphere, this spectrum was first established in simulations by Huang et al. (2001) and later confirmed by Sukoriansky et al. (2002) and Sukoriansky et al. (2007). The zonal and residual spectra in zonostrophic turbulence are shown in figure 5. Note that for \( n < n_\beta \), the energy in the zonal mode may exceed the energy in all other modes combined.

The parameter space of the zonostrophic turbulence regime is delineated by the chain inequality

\[
n_\epsilon \gtrsim 4n_\beta \gtrsim 8n_R \gtrsim 30, \tag{15}
\]

which yields \( R_\beta \gtrsim 2 \) (Sukoriansky et al. 2007). Note that if the small-scale forcing is associated with the baroclinic instability, then the forcing scale can be identified with the first deformation radius, \( L_D \). Taking into account that in this case, \( n_\epsilon = \pi R/L_D \), the inequality \( n_\epsilon \gtrsim 30 \) yields \( (L_D/R)^2 = Bu \lesssim 10^{-2} \). Generally, the inequality (15) instructs us that (i) the forcing acts on scales only weakly impacted by a \( \beta \)-effect; (ii) there exists a meaningful zonostrophic inertial range, \( n \in (n_R, n_\beta) \), and (iii) there exists a sufficient number of the lowest modes to resolve the large-scale friction processes and avoid the large-scale condensation. Note that although, formally, \( R_\beta \gtrsim 2 \) in the condensation regime, the small number of modes involved in the large-scale energy removal...
are unable to fully absorb the energy flux associated with the inverse cascade without distorting the latter and causing the energy saturation in the largest modes (Smith and Yakhot 1993, 1994, Sukoriansky et al 1999).

The zonal and residual spectra in the zonostrophic regime intersect at the introduced earlier transitional wavenumber,

\[ n_\beta = \left( \frac{C_\omega}{C_k} \right)^{3/10} \left( \frac{\beta_1^{3/5}}{\epsilon} \right)^{1/5} \simeq 0.5 \left( \frac{\beta_1^{3/5}}{\epsilon} \right)^{1/5}. \]  

(16)

To further elaborate the difference between the friction-dominated and zonostrophic regimes, it is instructive to analyze the Fourier-transform of the velocity autocorrelation function,

\[ U(\omega, m, n) = \frac{n(n+1)}{4} \left( |\psi_n^m(\omega)|^2 \right), \]  

(17)

where \( \psi_n^m(\omega) \) is a time Fourier transformed spectral coefficient \( \psi_n^m(t) \) defined in equation (9). In 2D turbulence without RHW, \( U(\omega, m, n) \) would be expected to acquire a symmetric bell shape around the zero frequency. When the waves are present, \( U(\omega, m, n) \) exhibits spikes at frequencies corresponding to the dispersion relation \( \omega = \omega_{m,n} \). The results of this analysis are shown in figures 6 and 7 for the friction-dominated and zonostrophic regimes, respectively. In the former case, for small wavenumbers, RHW produce sharp spikes almost exactly corresponding to the linear dispersion relation (10). Similar spikes also appear at higher wavenumbers although, being progressively broadened by turbulence, they are not as sharp. As evident from figure 6, RHW are present at \( n \) far exceeding both \( n_R \) and \( n_\beta \) thus indicating that there is no scale separation between the waves and turbulence and that both processes coexist on virtually all scales.

Figure 7 depicts more complicated case of wave–turbulence interaction characteristic of zonostrophic turbulence. As in figure 6, the wave spikes are sharp at small \( n \) and become progressively broadened by turbulence with increasing \( n \). In addition, some of the spikes are displaced from the positions stipulated by the linear dispersion relation (10). One also observes secondary spikes which are different

from RHW and are produced by nonlinear interactions. Similarly to figure 6, waves and turbulence coexist at wavenumbers exceeding both \( n_R \) and \( n_\beta \). An important result from the present study is that there exist neither a sharp transition between linear and nonlinear regimes nor a distinct spectral boundary between turbulence and RHW.

3. Zonostrophic turbulence in various environments

As shown in the previous section, a nondimensional parameter \( R_\beta = n_\beta/n_R \) plays a key role in determining the nature of a steady-state circulation of 2D turbulent flows with a \( \beta \)-effect. Using the balance equation between the small-scale forcing, \( \epsilon \), and the large-scale dissipation of the total kinetic energy, \( E_{\text{tot}} \), due to the linear drag,

\[ \epsilon = 2\lambda E_{\text{tot}}, \]  

(18)

one can recast \( R_\beta \) in terms of the basic internal,

\[ R_\beta = 2^{1/2} \left( \frac{C_\omega}{C_k} \right)^{3/10} \left( \frac{\beta U_5^5}{\epsilon^2} \right)^{1/10} \simeq 0.7 \left( \frac{\beta U_5^5}{\epsilon^2} \right)^{1/10}, \]  

(19)
or external, 

$$R_\beta = 2^{1/2} \left( \frac{C_Z}{C_K} \right)^{3/10} \left( \frac{\beta^2 \varepsilon}{\lambda^5} \right)^{1/20} \gtrsim 0.7 \left( \frac{\beta^2 \varepsilon}{\lambda^5} \right)^{1/20}$$ (20)

flow parameters (Sukoriansky et al 2007). Recall that the zonostrophic regime is characterized by $R_\beta > 2$ and the smallness of the Burger number, $Bu$. It is instructive to compare the values of $R_\beta$ for different environments. Table 1 compares a wide variety of environments with small $Bu$ that include the Coriolis turntable used in the Grenoble experiment (Read et al 2004, 2007), the terrestrial oceans, and the solar giant planets’ weather layers.

One can see that in the Grenoble experiment, the flow attained a marginally zonostrophic regime. As a result, the flow field exhibited zonation, spectral anisotropization, and the build-up of the zonal spectrum close to (14a).

Similarly to the Grenoble experiment, the oceanic flows also appear to be marginally zonostrophic. Visually, these flows are quite erratic and would rather be expected to belong in the transitional regime. Nevertheless, averaging in time reveals zonation (Maximenko et al 2005, Ollitrault et al 2006) and spectral anisotropization (Zang and Wunsch 2001) suggesting that $R_\beta$ is rather close to 2. In addition, some numerical models show the build-up of the barotropic zonal spectrum that agrees with (14a) (Galperin et al 2004). Note, however, that most of the cited oceanic data relates to the surface observations obtained with satellite altimetry which does not provide reliable estimate of the deep quasi-barotropic circulation under the thermocline. Such data of appropriate geographical coverage, duration and resolution is very difficult to obtain and our best hope to have a glance at the deep circulation at the present time would be using oceanic drifters (LaCasce and Bower 2000, Rossby 2007). Note also that even though the zonostrophic inertial range in the ocean is narrow and the barotropic currents are relatively weak, their considerable depth of penetration may suffice for these currents to have significant impact upon the large-scale dynamic and transport processes. Studies of these processes are now becoming an area of an active research (Eden 2006, Galperin et al 2004, Nadiga 2006, Ollitrault et al 2006, Richards et al 2006, Smith 2005).

For the solar giant planets, $R_\beta$ generally exceeds 10 thus indicating that their atmospheric circulations feature well established zonostrophic regimes. Indeed, the energy spectra of the zonal flows on these planets are consistent with equation (14a) in both the slope and the magnitude (Galperin et al 2001, Sukoriansky et al 2002).

### Table 1. Turbulence parameters in different environments.

<table>
<thead>
<tr>
<th>Environment</th>
<th>$n_z$</th>
<th>$n_\beta$</th>
<th>$n_R$</th>
<th>$Bu$</th>
<th>$R_\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coriolis turntable</td>
<td>40</td>
<td>8</td>
<td>7</td>
<td>$5 \times 10^{-4}$</td>
<td>0.5–2.3</td>
</tr>
<tr>
<td>Terrestrial oceans</td>
<td>400</td>
<td>55</td>
<td>40</td>
<td>$6 \times 10^{-3}$</td>
<td>1–2</td>
</tr>
<tr>
<td>Jupiter</td>
<td>1000</td>
<td>165</td>
<td>11</td>
<td>$3 \times 10^{-3}$</td>
<td>15</td>
</tr>
<tr>
<td>Saturn</td>
<td>500</td>
<td>135</td>
<td>8</td>
<td>$5 \times 10^{-5}$</td>
<td>17</td>
</tr>
<tr>
<td>Uranus</td>
<td>400</td>
<td>75</td>
<td>3</td>
<td>$5 \times 10^{-5}$</td>
<td>25</td>
</tr>
<tr>
<td>Neptune</td>
<td>500</td>
<td>70</td>
<td>3</td>
<td>$6 \times 10^{-5}$</td>
<td>25</td>
</tr>
</tbody>
</table>

4. Conclusions

Addressing the goals of the conference ‘Turbulent Mixing and Beyond,’ this paper presents a new flow regime, zonostrophic turbulence, characterizing large-scale circulations in planetary and terrestrial systems. It is shown that zonostrophic turbulence is a subset of geostrophic turbulence and is distinguished by a strongly anisotropic kinetic energy spectrum and slowly varying systems of alternating zonal jets. A parameter range conducive to the establishment of this regime is identified. The two most important parameters are the Burger number and the ratio, $R_\beta = n_\beta/n_R$, determining the width of the zonostrophic inertial range. More specifically, the zonostrophic regime develops when $Bu \lesssim 10^{-2}$ and $R_\beta \gtrsim 2$ simultaneously. Table 1 shows that while the former inequality is fulfilled in many natural systems, the latter one is not satisfied easily. Almost ideal natural laboratories for studies of the regime of zonostrophic turbulence present the solar giant planets where the zonostrophic inertial range is very wide and the regime is manifested via anisotropic spectrum and stable systems of alternating zonal jets (Galperin et al 2001, Sukoriansky et al 2002). Terrestrial oceans are only marginally zonostrophic (Galperin et al 2004). Attempts to reproduce this regime in the largest existing rotating tank, the Coriolis turntable, also yielded marginal zonostrophy at best (Read et al 2004, 2007). The difficulty in attaining wide zonostrophic inertial range can be understood by inspecting equation (20) which expresses $R_\beta$ via external flow parameters. Due to the very small power (1/20), one needs to vary these parameters by many orders of magnitude to achieve a small change in $R_\beta$. It is challenging, therefore, to obtain $R_\beta > 2$ in conventional fluids. Since the dependence on $\lambda$ in (20) is the strongest, it appears most promising to utilize for this purpose unconventional low viscosity fluids such as, for instance, the cryogenic helium (Niemela and Sreenivasan 2006). It is quite possible that a relatively small-scale cryogenic helium device such as the one used at ICTP in Trieste, Italy may yet prove to be the most effective test-bed for investigation of various processes taking place in the atmospheres of giant planets.

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